Lesson 10-1 Contents

Example 1

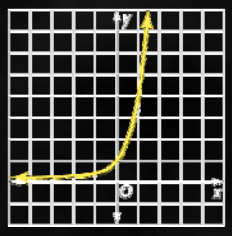
Sketch the graph of $y = 4^x$. Then state the function's domain and range.

Make a table of values. Connect the points to sketch a smooth curve.

x	$y = 4^x$
–2	$\frac{1}{16}$
–1	$\frac{1}{4}$
0	1
1	4
2	16

Example 1

Answer:

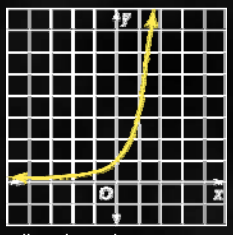


The domain is all real numbers, while the range is all positive numbers.

Your Turn

Sketch the graph of $y = 3^x$. Then state the function's domain and range.

Answer:



The domain is all real numbers; the range is all positive numbers.

Example 2a

Determine whether $y = (0.7)^x$ represents exponential *growth* or *decay*.

Answer: The function represents exponential decay, since the base, 0.7, is between 0 and 1.

Example 2b

Determine whether $y = \frac{1}{2}(3)^x$ represents exponential *growth* or *decay*.

Answer: The function represents exponential growth, since the base, 3, is greater than 1.

Example 2c

Determine whether $y = 10\left(\frac{4}{3}\right)^x$ represents exponential *growth* or *decay*.

Answer: The function represents exponential growth, since the base, $\frac{4}{3}$, is greater than 1.

Your Turn

Determine whether each function represents exponential *growth* or *decay*.

a. $y = (0.5)^x$ **Answer:** The function represents exponential decay, since the base, 0.5, is between 0 and 1.

b. $y = \frac{1}{3}(2)^x$ **Answer:** The function represents exponential growth, since the base, 2, is greater than 1.

c. $y = 10\left(\frac{2}{5}\right)^x$ **Answer:** The function represents exponential decay, since the base, $\frac{2}{5}$, is between 0 and 1.

Example 3a

Cellular Phones In December of 1990, there were 5,283,000 cellular telephone subscribers in the United States. By December of 2000, this number had risen to 109,478,000.

Write an exponential function of the form $y = ab^x$ that could be used to model the number of cellular telephone subscribers y in the U.S. Write the function in terms of x, the number of years since 1990.

For 1990, the time x equals 0, and the initial number of cellular telephone subscribers y is 5,283,000. Thus the y-intercept, and the value of a, is 5,283,000.

For 2000, the time x equals 2000 – 1990 or 10, and the number of cellular telephone subscribers is 109,478,000.

Example 3a

Substitute these values and the value of a into an exponential function to approximate the value of b.

$y = ab^x$ Exponential function

$109{,}478{,}000 = 5{,}283{,}000b^{10}$ Replace x with 10, y with 109,478,000 and a with 5,283,000.

$20.72 \approx b^{10}$ Divide each side by 5,283,000.

$\sqrt[10]{20.72} \approx b$ Take the 10th root of each side.

Example 3a

To find the 10th root of 20.72, use selection $5:\sqrt{}$ under the MATH menu on the TI-83 Plus.

Keystrokes: 10 MATH 5 20.72 ENTER 1.354063324

Answer: An equation that models the number of cellular telephone subscribers in the U.S. from 1990 to 2000 is $y = 5{,}283{,}000(1.35)^x$.

Example 3b

Suppose the number of telephone subscribers continues to increase at the same rate. Estimate the number of US subscribers in 2010.

For 2010, the time x equals 2010 – 1990 or 20.

$y = 5{,}283{,}000(1.35)^x$ Modeling equation

$y = 5{,}283{,}000(1.35)^{20}$ Replace x with 20.

$y \approx 2{,}136{,}000{,}000$ Use a calculator.

Answer: The number of cell phone subscribers will be about 2,136,000,000 in 2010.

Your Turn

Health **In 1991, 4.9% of Americans had diabetes. By 2000, this percent had risen to 7.3%.**

a. Write an exponential function of the form $y = ab^x$ could be used to model the percentage of Americans with diabetes. Write the function in terms of x, the number of years since 1991.

Answer: $y = 4.9(1.05)^x$

b. Suppose the percent of Americans with diabetes continues to increase at the same rate. Estimate the percent of Americans with diabetes in 2010.

Answer: 11.4%

Example 4a

Simplify $5^{\sqrt{3}} \div 5^{\sqrt{2}}$.

Answer: $5^{\sqrt{3}} \div 5^{\sqrt{2}} = 5^{\sqrt{3}-\sqrt{2}}$ Quotient of Powers

Example 4b

Simplify $\left(6^{\sqrt{5}}\right)^{\sqrt{5}}$.

$\left(6^{\sqrt{5}}\right)^{\sqrt{5}} = 6^{\sqrt{5} \cdot \sqrt{5}}$ Power of a Power

Answer: $= 6^{\sqrt{30}}$ Product of Radicals

Your Turn

Simplify each expression.

a. $2^{\sqrt{5}} \div 2^{\sqrt{3}}$

Answer: $2^{\sqrt{5}-\sqrt{3}}$

b. $\left(7^{\sqrt{3}}\right)^{\sqrt{7}}$

Answer: $7^{\sqrt{21}}$

Example 5a

Solve $4^{9n-2} = 256$.

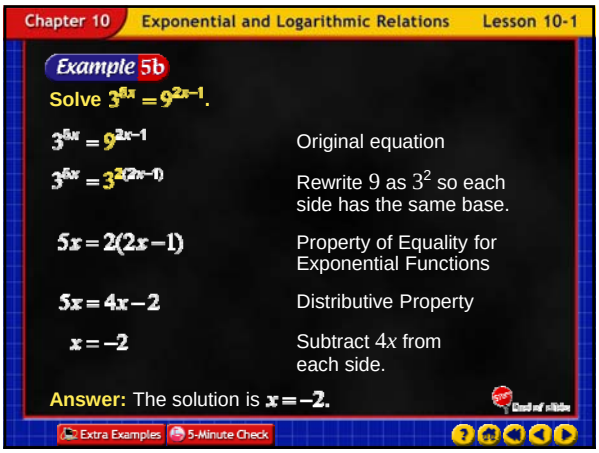
$4^{9n-2} = 256$	Original equation
$4^{9n-2} = 4^4$	Rewrite 256 as 4^4 so each side has the same base.
$9n - 2 = 4$	Property of Equality for Exponential Functions
$9n = 6$	Add 2 to each side.
$n = \frac{2}{3}$	Divide each side by 9.

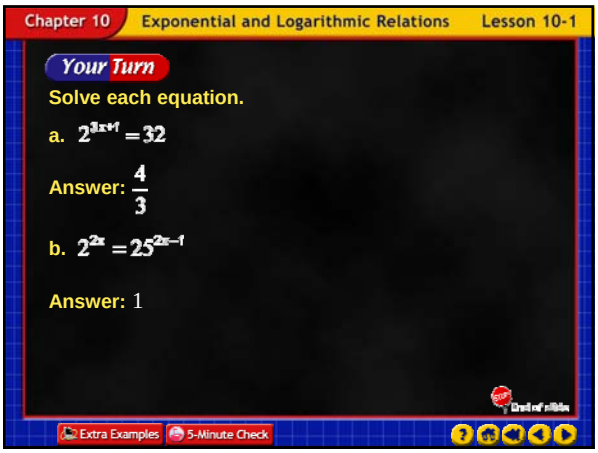
Answer: The solution is $n = \frac{2}{3}$.

Example 5a

Check

$4^{9n-2} = 256$	Original equation
$4^{9\left(\frac{2}{3}\right)-2} \stackrel{?}{=} 256$	Substitute $\frac{2}{3}$ for n.
$4^{4} \stackrel{?}{=} 256$	Simplify.
$256 = 256$ ✓	Simplify.


Chapter 10 Exponential and Logarithmic Relations Lesson 10-1
Example 5b
Solve $3^{5x} = 9^{2x-1}$.
$3^{5x} = 9^{2x-1}$ Original equation
$3^{5x} = 3^{2(2x-1)}$ Rewrite 9 as 3^2 so each side has the same base.
$5x = 2(2x-1)$ Property of Equality for Exponential Functions
$5x = 4x - 2$ Distributive Property
$x = -2$ Subtract $4x$ from each side.
Answer: The solution is $x = -2$.
Extra Examples
5-Minute Check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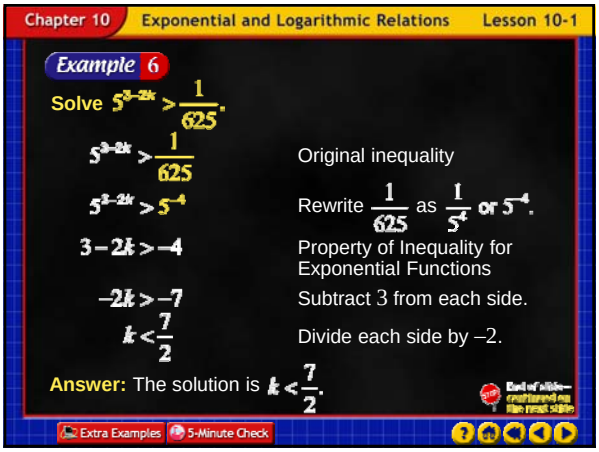
Your Turn
Solve each equation.
a. $2^{3x+1} = 32$
Answer: $\frac{4}{3}$
b. $2^{2x} = 25^{2x-1}$
Answer: 1
Extra Examples
5-Minute Check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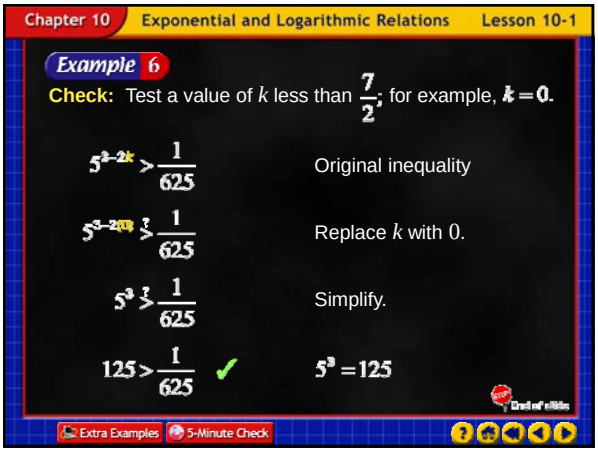
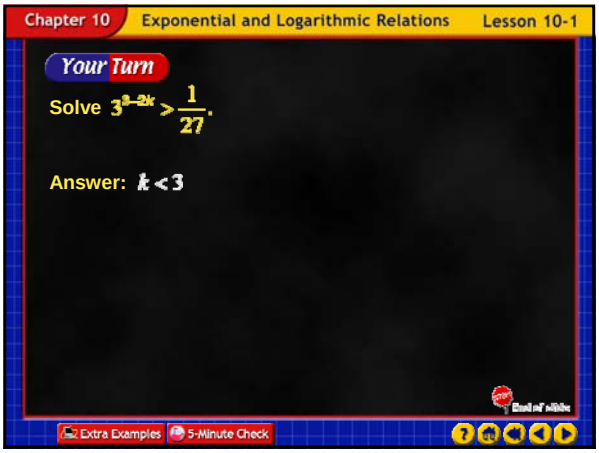
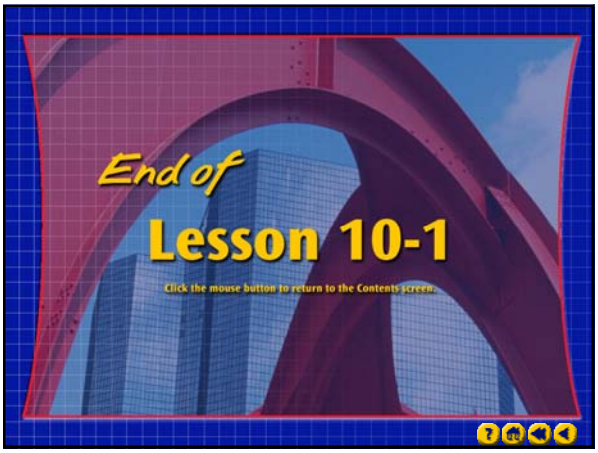
Example 6
Solve $5^{3-2k} > \frac{1}{625}$.
$5^{3-2k} > \frac{1}{625}$ Original inequality
$5^{3-2k} > 5^{-4}$ Rewrite $\frac{1}{625}$ as $\frac{1}{5^4}$ or 5^{-4}.
$3 - 2k > -4$ Property of Inequality for Exponential Functions
$-2k > -7$ Subtract 3 from each side.
$k < \frac{7}{2}$ Divide each side by –2.
Answer: The solution is $k < \frac{7}{2}$.
Extra Examples
5-Minute Check


Chapter 10 Exponential and Logarithmic Relations Lesson 10-1
Example 6
Check: Test a value of k less than $\frac{7}{2}$; for example, $k = 0$.
$5^{3-2k} > \frac{1}{625}$ Original inequality
$5^{3-2(0)} \stackrel{?}{>} \frac{1}{625}$ Replace k with 0.
$5^3 \stackrel{?}{>} \frac{1}{625}$ Simplify.
$125 > \frac{1}{625}$ ✓ $5^3 = 125$
Extra Examples
5-Minute Check


Chapter 10 Exponential and Logarithmic Relations Lesson 10-1
Your Turn
Solve $3^{3-2k} > \frac{1}{27}$.
Answer: $k < 3$
Extra Examples
5-Minute Check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Lesson 10-1
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